A guide to explore the Pierre Auger Observatory public data

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Abstract

This note presents a guide for exploring the Auger public data developed for students in the final years of high school (15 to 18 years old). It includes the guide itself, a section with proposed answers meant as a guideline for teachers or tutors, and introductory remarks on the scope of the guide and on the experience of using it in Portugal for a few years already. The guide is to be used together with the material available in the Auger Public Event Display and Education WEB pages, making the exploration of the public data more fruitful and complete.

Foreword

The guide “Questions and answers on Extreme Energy Cosmic rays - A guide to explore the Pierre Auger Observatory public data” was developed by the Auger LIP team and has been used in Portugal since 2009 in the framework of the programme “Scientific occupation of young people on holidays” of the outreach agency “Ciência Viva”. Within this programme, students spend one to two weeks in a research laboratory, LIP in this case, developing activities related to one of the projects of the laboratory. The Auger LIP group hosted 4 to 6 students each year, proposing them this guide as a way to learn about high energy cosmic rays and their detection, and to analyse the data made publicly available by the Pierre Auger Observatory. During these periods, the students were working at LIP and had direct access to support from researchers. This year, the guide is proposed to be used in schools for activities to be developed along the year. In this situation, direct support to the students will be given by their teachers, and the activities will be distributed for longer periods of time. Proposed answers and discussion topics are added as a tool for teachers and tutors. Seminars of the LIP researchers in the schools and regular meetings between the teachers and the researchers are also foreseen. Both in the school and in the research lab context, a practical demonstration of the existence of cosmic rays and cosmic showers and their detection are performed, as well as introductory talks to the subject. At LIP, both a demonstrantion spark chamber and the small scintillator array installed at IST (Technical University) were used for this purpose. If, in addition, a laboratory activity can be proposed, it will be a valuable complement to these data analysis acitvities.
The guide is organized as follows. In Section 1 the cosmic ray spectrum is introduced and explored. This “warming up” section deals with orders of magnitude, units, solid angles, fluxes. It is in general perceived as relatively “easy” by students. Section 2 introduces basic notions of fundamental particles and interactions, and explores simple Heitler-like models to describe the shower development. This is the more formal section, requiring some mathematical background (logarithms, exponentials, trigonometry). In fact, this section of the guide nicely matches the maths program of the final years of high school, and can be regarded as an interesting and well motivated set of exercices. Students with the adequate background in mathematics usually appreciate the logical, step by step understanding of showers. Section 3 comes in like a short break - an opportunity to rest while learning about the Pierre Auger Observatory and exploring its WEB site. The concept of sampling and the detection techniques are introduced. Sections 4 and 5 are the ones that directly explore the Pierre Auger Observatory public data, available in the “Public Event Explorer” WEB page. While in section 4 a shower-by-shower analysis is proposed, statistical studies and distribution analyses are carried out in section 5. Data analysis was usually performed using Excel, except in a few cases of more

advanced students who were familiar with ROOT or for some reason wanted to learn about it. In Section 4, the goal is that students understand the basics of air shower reconstruction, without getting into complex algorithms: which quantities are measured directly in each station? From there, how are the direction and the core location found? What can give us a hint of the primary energy? This was often considered the most motivating and challenging section. The goals and the required level of guidance very much depend on the profile of the students. In section 5 we get closer to the physics results of Auger and their implications, The spectrum and the arrival directions are the main ingredients. However, the statistics and the energy range are obviously limited. Some simple but realistic calculations are proposed and help to understand and discuss the results.

Acknowledgements

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We thank the high school students that, through their enthusiastic participation in the internships, have tackled the difficulties and provided valuable suggestions that have greatly improved this guide. In particular we thank the participants in the internship of summer 2009: Miguel Rodrigues, Mariana Santos, Ricardo Sousa, Rui Trindade; the participants in summer 2010: Tiago Silva, Emanuel Sousa; the participants in summer 2011: Pedro Baião, André Graça, Thomas Pimenta, Guilherme Rato; and the participants in the internship of summer 2012: José Cordeiro, Miguel Jacques, Rui Marques, Filipe Matos, João Pedro, Tomás Reis.
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Questions and Answers on Extreme Energy Cosmic Rays

- A guide to explore the public data of the Pierre Auger Observatory -

Since 100 years, we know that planet Earth is constantly hit by particles arriving to us from the cosmos. Such particles have very diverse energies, abundances and origins, and many questions remain to be answered about them. This study is devoted to extreme energy cosmic rays - the rarest, most energetic, particles arriving to us from the Universe. When these particles reach the top of the atmosphere, they produce a shower of millions of particles: the higher the energy of the initial particle, the larger the number of particles in the shower.
The Pierre Auger Observatory detects these particle showers with the goal of solving some of the mysteries of extreme energy cosmic rays: what are these particles? Where do they come from? How are they produced and accelerated to such high energies? The Pierre Auger Observatory decided to make 1% of the data it is collecting since 2004 available to all those who wish to learn more about extreme energy cosmic rays. These data are available on the "Public Event Explorer" web page, which is updated every day.
This guide is meant to be a roadmap for your exploration of the Auger public data, making it more fruitful and complete. The work is organised in the following parts:

1. The cosmic ray spectrum
2. How do cosmic ray showers develop
3. How do we detect cosmic ray showers
4. How do we measure extreme energy cosmic rays
5. What are these data telling us about extreme energy cosmic rays

Useful links:

- The Pierre Auger Observatory: http://www.auger.org
- Public Event Explorer (Auger public data page): http://auger.colostate.edu/ED
- Auger education page: http://www.auger.org/education/Auger_Education
- Particle Physics basics: http://www.particleadventure.org

1) The cosmic ray spectrum

The graph in figure 1 shows what we call the energy spectrum of cosmic rays - for each energy interval (in the X axis) we have (in the Y axis) the differential flux of particles: the average number of particles with this kinetic energy which, at the top of the atmosphere, reach each area of 1 m^2 during the time interval of 1 second, and with an arrival direction within an angle in space (solid angle) of 1 stereo-radian (sr)[1] per GeV of energy. Notice that both the X and the Y axes are in logarithmic scale[2]! In high energy physics we

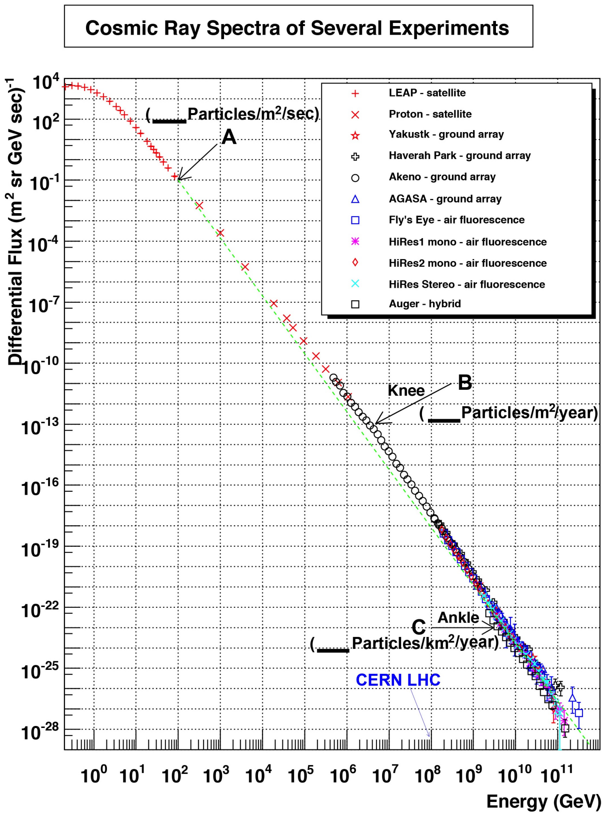
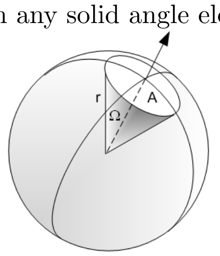


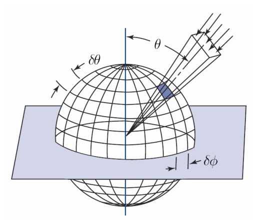
Figure 1: The cosmic ray energy spectrum.

normally use as energy unit the electron-Volt (eV), defined as the energy of a particle with the charge of the electron under an electric potential difference of 1 Volt; 1 GeV= 10^9 eV. As the charge of the electron is $q_e = -e = -1.6 \times 10^{-19}$ C, we have 1 eV= 1.6×10^{-19} J, and we conclude that 1 Joule corresponds to about 10^{19} eV= 10^{10} GeV.
Note that the flux is defined as the number of particles arriving per unit of time (Δt) and per unit of area. What is shown in the graph is in fact the variation of the flux with energy, per unit of energy (in GeV) and per unit of solid angle (in sr): $\frac{1}{\Delta\Omega} \cdot \frac{\Delta \mathrm{F}}{\Delta \mathrm{E}}$. To estimate the average flux in a given energy region, we need to multiply this flux variation

by the width ΔE of the energy interval to which corresponds the energy value we are considering in the graph. For the present case you can just take: $\Delta E \simeq E$. For flat detectors such as the Pierre Auger Observatory, we must also multiply by the effective angular acceptance of the detector Ω_{eff} (which is reasonable to take as 1/4 of the total solid angle[1]).
This graph puts together measurements performed in many experiments worldwide in the course of several decades. The cosmic rays with energies below about 1 GeV (10^9 eV) arrive mostly from the Sun. Above this energy they are originated somewhere outside de solar system. As for the highest energy cosmic rays (above roughly 3×10^{18} eV), we think they come from outside our galaxy. We will come back to this question in the final part of this guide.

[1] The solid angle is the 3-dimensional equivalent to the angles we are used to in the plane. If (see figure) a cone contains a portion A of the surface of a radius r sphere, the solid angle inside the cone Ω is given by $\Omega = A/r^2$. The solid angle is measured in stereo-radians. Given that the total area of the surface of a sphere is $A = 4\pi r^2$, the total solid angle is 4π. The areas in a sphere can be computed in spherical coordinates (r, θ, ϕ) (see figure) as $r^2 \text{sen}\theta d\theta d\phi$, where $d\theta$ and $d\phi$ are the angular widths of the area. Note that a solid angle interval can be written as $d\Omega = \text{sen}\theta \times d\theta \times d\phi = d(cos\theta)d\phi$. It is also worth noting that isotropy means equal probability of coming from any solid angle element.





[2] In physics we deal with very different orders of magnitude, to which correspond different prefixes to be added to the name of the unit. For example, for distances:
nanometer [nm] = 10^{-9} m
micrometer [μm] = 10^{-6} m
milimeter [mm] = 10^{-3} m
meter = 1 m
kilometer [km] = 10^3 m
megameter [Mm] = 10^6 m
gigameter [Gm] = 10^9 m
terameter [Tm] = 10^{12} m
petameter [Pm] = 10^{15} m
exameter [Em] = 10^{18} m
1 pc = 3.26 light-years
1 light-year= 9.46×10^{15} m

Logarithms are the inverse of powers: the base 10 logarithm of a number x, $\log_{10}(x)$, is the number 10 should be raised to in order to obtain x. In a logarithmic scale of base 10, $\log_{10}$, we measure orders of magnitude: each division in the scale corresponds to ten times more than the previous one. For example, if one dash in the scale corresponds to $0.1 = 10^{-1}$, the next one will be $1 = 10^0$ and the next one $10 = 10^1$. We can also use $\log_2$ to measure powers of base 2. More widely used, and reffered to simply as log or ln, is the natural logarithm, which has for base the Neper number, $e = 2.71828...$ Its inverse function, the exponential, has numerous applications in the modeling of phenomena in very diverse science fields. [Note that: $e^{\log 2} = 2$; $2^{1/\log 2} = e$; $\log(2^a) = a \cdot \log 2$; $\log_a b = \log(a)/\log(b)$]

Questions:

1.1) 1 Joule is the order of magnitude of the kinetic energy of one apple (about 100 g) falling from a tree (a height of roughly 1 m). A cosmic ray of 10^{19} eV is a tiny particle (let's say, a proton) in which 1 Joule of energy is concentrated!
How many protons +neutrons (total number) are there inside an apple?
How does the average kinetic energy per proton/neutron compare in the two cases (apple and cosmic ray)?
Hint: Consider an apple is made of protons and neutrons only, and the masses of protons and neutrons to be the same (Avogrado's number is $N_A = 6.022 \times 10^{23}$).
1.2) What comment would you make about the inclination of the cosmic ray energy spectrum? Assuming it is approximately a straight line between **A** and **C**, what is the slope?
1.3) What is the number of particles per unit of area and time arriving to a detector on Earth for the energies indicated in the graph by the arrows:
A) 10^{11} eV;
B) 3×10^{15} eV ("knee" region);
C) 3×10^{18} eV ("ankle" region)?
Using the values you just computed, fill in the white spaces marked in the graph. No exact values are required, just the order of magnitude. **Mind the units!**
Try to represent your results in graph form using linear scales and discuss the advantages of log scales.
1.4) The AMS detector at the International Space Station has unique capabilities to characterise the cosmic rays reaching the Earth, for energies up to 1 TeV (10^{12} eV). Considering that the effective acceptance of the detector is 0.5 m^2 sr, estimate the number of particles with an energy of 1 TeV that cross the detector in one year.
Taking into account the flux of cosmic rays above 10^{19} eV, what area should a detector have in order to record about 100 such particles per year? Repeat the exercice for 10^{20} eV.
1.5) How do these areas compare with the dimensions of the Pierre Auger Observatory? Find the information you need on the public data web page. Using the tool available there, compare the dimensions of the Auger detector in the Argentinian Pampa with regions of the Planet more familiar to you.
How can one instrument such large areas?
1.6) Why are there so few data points in the energy range between **A** and **B**?

2) How do cosmic ray showers develop

High energy cosmic rays reaching the top of Earth's atmosphere interact with the air molecules, producing showers (or cascades) with millions of particles. It is a part of this shower that we are going to measure in cosmic ray detectors at ground. The fact that we do not detect the initial particle makes the measurement of its properties an indirect one. However, showers are presently the only way to study cosmic rays of such high energies: as seen in the previous section, the fluxes are extremely low, and it is simply not feasible to place in the upper atmosphere, or to send to orbit, detectors which are large enough to collect a reasonable number of events. We are thus left with the possibility of observing these particles after they interact in the atmosphere, building detectors able to measure their properties in the best possible way, in the largest possible areas.
Having a clear picture of how showers develop, what kind of particles they are made of and which are the energies of these particles, is essential for the work that will follow in the next sections: to understand how showers are detected and how can we reconstruct the properties of the particles that originated them.

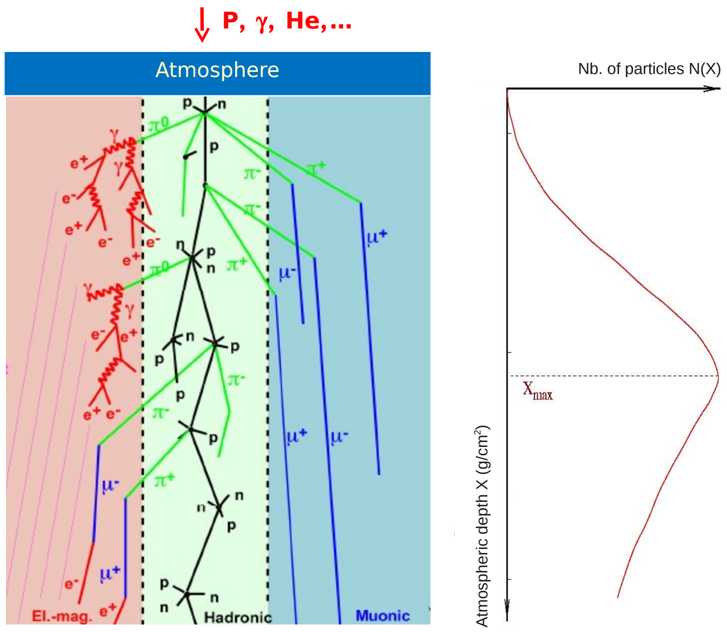


Figure 2: Longitudinal development of a shower in the atmosphere. The three different areas are in reality superimposed in space and develop simultaneously.

In a cascade initiated by a proton or a nucleus, there are a few, very high energy interactions, from which results a set of particles that will originate the 3 basic components of the cascade (see figure 2):

- The nuclear fragments, protons, neutrons and other hadrons[3] continue a series of interactions dominated by the strong nuclear force and form the bulk of the hadronic cascade, which develops in the central region, near the shower axis;
- Neutral pions (π^0) decay almost instantly into pairs of photons; each photon will then convert into an electron-positron pair, which will then radiate photons, and so on, originating the electromagnetic component of the shower;
- Charged pions ($\pi^\pm$), which contribute to the hadronic component of the cascade when they interact with the atmosphere, may also decay. In this case they originate neutrinos (which are not detected in this type of observatory) and muons, which have a low interaction probability and a lifetime large enough to be able to reach the ground.

In a cascade initiated by an electron, positron or photon (particles with electromagnetic interaction only), the electromagnetic component will be largely dominant.
What will happen to each particle in the cascade (will it decay? Will it interact through one of the several competing processes?) depends essentially on the particle type and energy. The "story in 3 components" results from the fact that we know the most probable outcome for the most abundant particle types at the typically involved energies.
The number of particles in the cascade does not increase forever: when the energy of the particles drops below a certain value they no longer lead to the production of new particles and eventually are absorbed. The maximum of the cascade occurs approximately when the average particle energy becomes lower than this value of critical energy. Only a small fraction of the particles reaches the ground - how many? This will depend on the type of incident particle, its energy and direction, and on the altitude of the detector site.
The development of air showers is a complex process, and its study requires time-consuming computer simulations, or solving complicated equations. There are however simple models able to provide realistic estimates for some of the most relevant quantities, which are very useful: they give cosmic ray physicists the possibility of having a quick and intuitive idea of the results expected in experiments or in more complex calculations, bringing to light the main aspects of the relevant physics mechanisms. This is the case of the Heitler model for electromagnetic showers, and of its generalisations for showers initiated by protons and nuclei.

Many of the Auger data analyses have the goal of determining the nature of the primary particles. Are they protons? Are they iron nuclei? Or a mixture of several species? Among the shower variables that may help to disentangle the different types, are the depth at which the shower reaches its maximum number of particles and the number of muons reaching the ground. We will now see that, with these simple models, we can make relevant and realistic predictions for the behaviour of these observables!
Let us start by noticing that the probability for a particle to interact with an atmospheric molecule depends on the probability that the two collide - and thus on the atmospheric density. And we know that the density is far from being the same near ground, at 10 km,

or at 40 km altitude! What really matters is not the distance travelled by the particle but the "amount of atmosphere" it crosses. For this reason we normally use, instead of the altitude, the quantity X, atmospheric depth, which corresponds to the total atmospheric density crossed by the particle, measured in g/cm^2 [4].

Questions:

2.1) What is the weight in kg of the atmosphere above your head (you can consider an area of 20×20 cm^2) at sea level and at the top of the Aconcagua mountain (Andes, province of Mendoza, Argentina, altitude 6962 m)?
The "amount of atmosphere" crossed by a cosmic particle depends, obviously, on the behaviour of the atmospheric density as a function of the altitude (density profile) which can vary with time and space. And what if the particle comes in almost horizontaly (nearly paralel to ground)? How does the "amount of atmosphere" depend on the angle of the incident particle with the vertical of the place where it would hit ground?
2.2) In a first step, we assume that our shower is initiated by a photon - a particle having only electromagnetic interaction. Let E_0 be the energy of the primary particle. And let us consider that the electrons, positrons and photons in the cascade always interact after travelling a certain atmospheric depth d, and that the energy is always equally shared between the two particles. With this assumptions, we can schematically represent the cascade as in figure 3(a).

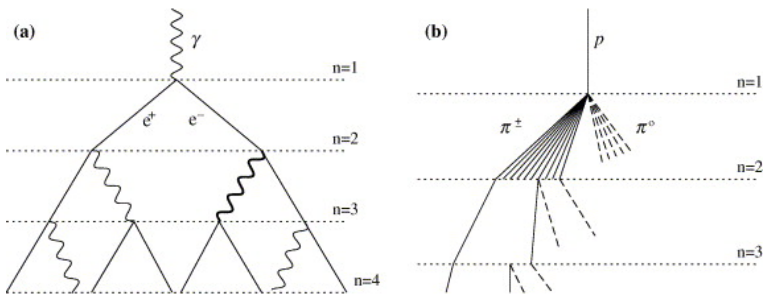


Figure 3: Schematic representation of Heitler-like models for the longitudinal development of electromagnetic (a) and hadronic (b) air showers.

For a number of interactions $n = 1, 2, 3, ...$ fill in the table, giving the depth X at which

[3] Hadrons are particles made of quarks and/or anti-quarks and thus subjected to the strong nuclear force. For more on elementary particles, have a look at the Useful Links in the beginning of this guide.

[4] It should be noted that if we cross an atmospheric layer of thickness δ low enough for density to be approximately constant (ρ), the atmospheric depth X, the "amount of atmosphere" crossed, is given by $X = \rho \times \delta$. For a large thickness of atmosphere, we will have to add up the X values obtained in many small sub-layers of approximately constant density (in the limit of infinitesimal thickness layers, this in an integration!) Thus, and considering that density can be expressed in g/cm^3 and thickness has units of length, X can be expressed in g/cm^2.

the interaction occurs, the number of particles N in the shower and the energy E of these particles.
Can you find analytical expressions for the number of particles and for the energy of each particle at depth X as a function of X, d, n and E_0?

n	X	N	E
0	0	1	E_0
1	d	2	$E_0/2$
2	2d	4	$E_0/4$
3			
4			
...			

2.3) But the shower does not develop forever! Below a critical energy E_c, other interaction processes with the atmosphere, which do not contribute with new particles to the cascade, become dominant.
Using the expressions you just obtained, and considering the atmospheric depth where the shower reaches its maximum, X_{max}, write expressions for:
(i) the particle energy,
(ii) the number of particles N_{max},
(iii) the atmospheric depth X_{max},
as a function of E_0, E_c and $\lambda = d/(ln2)$ (see [2]).
Compare the way (slower or faster) how N_{max} and X_{max} vary with the energy of the primary particle.
2.4) Estimate X_{max} and N_{max} for a shower originated by a primary cosmic ray with energy
$E_0 = 10^{19}$ eV, using $E_c = 85$ MeV and $\lambda = 36.7$ g/cm^2.
We know that a vertical shower with this energy reaches its maximum very near (or even after!) the ground in Auger. Find out which is the value of X corresponding to the Observatory and check in this way if your prediction is approximately correct.
2.5) Enumerate some factors that could complicate the picture, leading to deviations with respect to this simplistic model.
2.6) Let us now consider a shower initiated by a proton of energy E$_0$. We will describe it with the simple model of figure 3(b): after each depth d an equal number of pions (take $n_\pi = 10$) of each of the 3 types is produced: π^0, π^+, π^-. Neutral pions decay and their energy is transfered to the electromagnetic cascade. Only the charged pions will feed the hadronic cascade. We consider that the cascade ends when these particles decay as they reach a given decay energy E_{dec}, after n interactions, originating a muon (plus an undetected neutrino).
For a number of interaction $n = 1, 2, 3, ...$ fill in the table, giving the total number of particles in the shower N_{tot}, the number of charged particles N_{ch} and the energy of each particle E and the sum of the energies of all charged particles E_{ch}.
When reaching an energy E_{dec}, charged pions decay originating muons and neutrinos. After how many interactions does this occur?

Within this model, derive a prediction for the number of muons produced in the cascade N_μ as a function of E_0 and E_{dec}.

n	N_{tot}	N_{ch}	E	E_{ch}
0	1	1	E_0	E_0
1	30	20	$E_0/30$	$2E_0/3$
2				
3				
4				
...				

2.7) Estimate the percentage of the total energy that is transferred to the electromagnetic shower, for $E_0 = 10^{19}$ eV and knowing that $E_{dec} \simeq 20$ GeV.
2.8) Finally, let us consider a shower initiated by an iron nucleus (mass number $A = 56$). We assume the validity of the superposition principle, according to which a nucleus of mass number A and energy E_0 behaves like A nucleons of energy E_0/A.
Derive expressions for:
i) the number of particles at the shower maximum
ii) the depth where this maximum is reached,
iii) the number of muons produced in the shower,
and relate them to the equivalent equations obtained for proton initiated showers. How do these observables change?
2.9) Let us now consider the lateral distribution of the shower particles on ground. The shower particles do not all travel along exactly the same direction as the primary particle, but most do have similar directions.
Looking at the lateral profile of the event in figure 4 (a nearly vertical shower available in the public data web page) estimate the size on ground of these highest energy showers. Compute the angle with the shower axis of a muon produced 10 km above ground and detected at such a distance from the shower core.
2.10) In reality, the lateral spread of the cascade is caused mainly by multiple dispersions of the charged particles in the Coulomb field of the atmospheric nuclei. A useful parameter to characterise the lateral shower development in a way independent from the crossed medium is the Moliére radius: $r_M = E_s \times \lambda / E_c$, with $E_s \sim 21$ MeV. About 90% of the shower particles are contained inside a cylinder around the shower axis with radius equal to the Moliére radius.
Compute the Moliére radius in Auger using the values given in this guide and the atmospheric density corresponding to the altitude of the Observatory (see figure 5) to convert the value to units of distance.

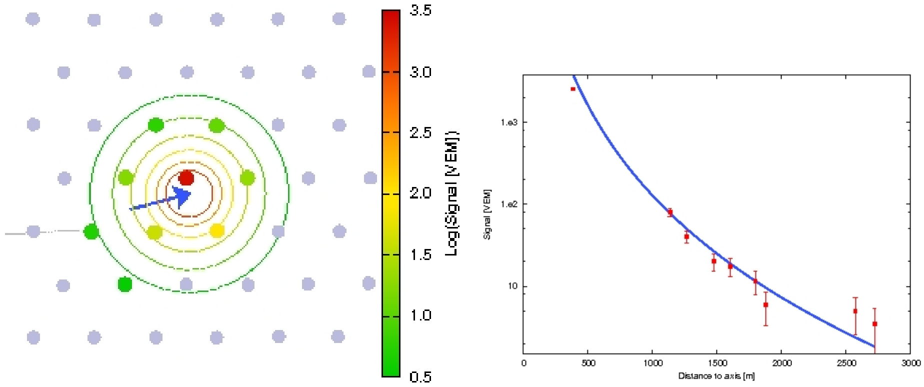


Figure 4: Lateral profile and ground distribution of the shower particle density for the shower with ID 10998100.

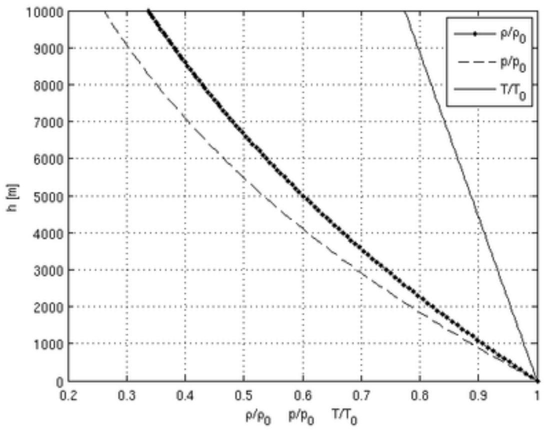


Figure 5: Atmospheric profiles. Reference values: $T_0 = 20°$ C, $p_0 = 1$ atm $=$ 101300 Pa, $\rho^0 = 1.2$ kg/m^3.

3) How do we detect cosmic ray showers

The Pierre Auger Observatory combines two independent techniques to detect and characterise cosmic ray showers: one is based on the surface detector, that collects information on the charged particles reaching ground; the other is based on the fluorescence detector - it detects light produced in the atmosphere due to the passing of the cascade particles. While the surface detector is always in operation, this fluorescence detector collects data only on dark, moonless nights. In this guide we will concentrate on the surface detector, since the data made public by the Observatory were collected using the surface detector.
The Auger surface detector consists of more than 1600 water tanks, sketched in figure 6, placed at about 1.5 km from each other, and which will sample the charged particles of the shower as they reach ground.

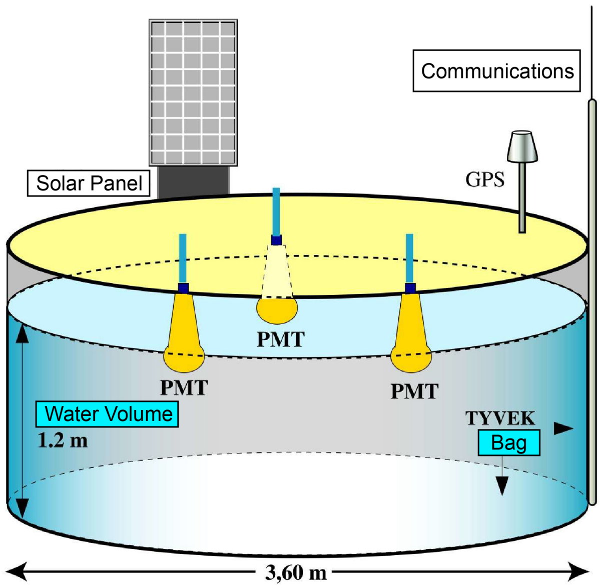
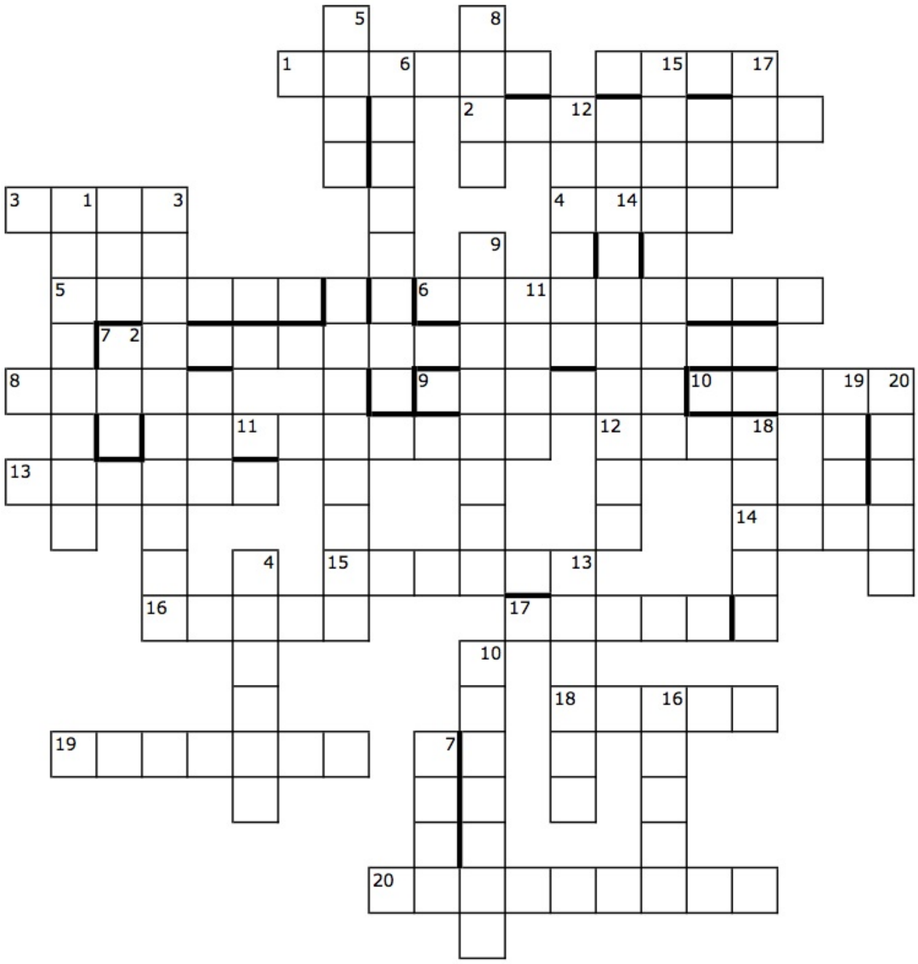


Figure 6: Water tank of the Pierre Auger Observatory.

The shower particles reaching ground are detected in the tanks due to the Cherenkov effect: when a charged particle travels at a speed above the speed of light in the medium (the water inside the tank) - something that is not possible in vacuum (why?) - they will emit Cherenkov radiation. This light will be detected by 3 photomultipliers (light detectors, PMT). The photons are emitted while the particle crosses the tank (or until it is absorbed by the water in the tank) and many of them quickly reach the photomultipliers, which convert them, by photoelectric effect, in a measurable electric current. The collected electric signal is proportional to the number of charged particles crossing the tank, and

can thus be used to reconstruct the particle density at the spot where the tank is located (see figuree 6 for the tank area).
The more energetic the shower is, the larger is the number of particles reaching ground. The number of particles per m^2 (particle density) in each tank is also a function of the distance of the tank to the shower core (the point on ground the primary particle would hit if it would not interact and produce a shower). Using the particle densities per tank we can estimate the core location and the total energy of the shower - and of the primary particle. Finally, the arrival times of the particles to the tanks depend on the arrival direction of the shower. The signals collected in the tanks thus allow to estimate the arrival direction and the energy of the primary particle.



With this information, and more you can find on the web, (in particular in the Pierre Auger Observatory site) complete the grid with the appropriate words:

Horizontal - Lines (number in the top left)
1. Object of study at the Pierre Auger Observatory in 2 words
2. Town of the Pierre Auger Observatory
3. Point of impact of the shower axis on ground
4. Approximate number of years since cosmic ray discovery (in spanish)
5. Property of the primary cosmic ray related to signal size and number of tanks
6. Property of the primary cosmic ray obtained from the times measured in the tanks
7. Name of the "PMT", device that converts light signals into an electric current
8. It is eliptical for a shower footprint on ground
9. Family name of one of the founders of the Pierre Auger Observatory
10. Landscape where the Pierre Auger Observatory is located
11. One of the 4 Fluorescence detector eye locations (no article)
12. First name of one of the founders of the Pierre Auger Observatory (inverted)
13. This particle constitutes the majority of cosmic ray primaries (inverted)
14. Name of the feature in the cosmic ray energy spectrum located around 1 PeV
15. One of the 4 fluorescence detector eye locations (no article)
16. Part of the Auger array with tanks at smaller distances and underground muon detectors
17. Commonly used acronym for the ultra high energy cosmic rays studied in Auger
18. Medium inside the tanks that allows charged particles to be detected
19. Province where the Pierre Auger observatory is located
20. Effect used for charged particle detection, when they travel faster than light in a medium

Vertical - Columns (number in the top right)
1. One of the 4 fluorescence detector eye locations (inverted)
2. Initials for the Pierre Auger Observatory
3. Large medium over ground that allows the detection of secondary particles (inverted)
4. First name of the scientist that proved the extra-terrestrial nature of cosmic rays
5. One of the 4 fluorescence detector eye locations (2 words)
6. Common name of the energy distribution of the cosmic ray flux (without the word "energy")
7. Family name of the scientist refered in number 4 above (inverted)
8. Most important measurement obtained with the GPS systems placed on top of the tanks
9. Gas molecule that produces the fluorescence light detected by the fluorescence detector
10. All these particles are detected in the tanks
11. There are many (some dry) in the region where the observatory in located (in spanish)
12. When going to take notes in the field, such a device is very handy (inverted)
13. Name of the set of secondary particles created from the primary cosmic ray
14. These cosmic rays traverse much more atmosphere than vertical ones
15. Country where the Pierre Auger Observatory is located
16. Reflective membrane covering the inner wall of the tanks
17. Origin of the energy powering the tanks
18. Name of the feature in the cosmic ray energy spectrum around 1 EeV
19. Centre of the sky region not accessible to the Pierre Auger Observatory (north...)
20. Mountain chain very close to the Observatory

Questions:

3.1) The area covered by the Observatory is of the order of 3000 km^2. Assuming the simple polygonal geometry in figure 7, compute the total area. All angles between the black lines (horizontal and vertical) and the white edges of the polygon are 30°, 45°, 60°, 90° or 120° (except for one 15° angle and the corresponding 75° angle).

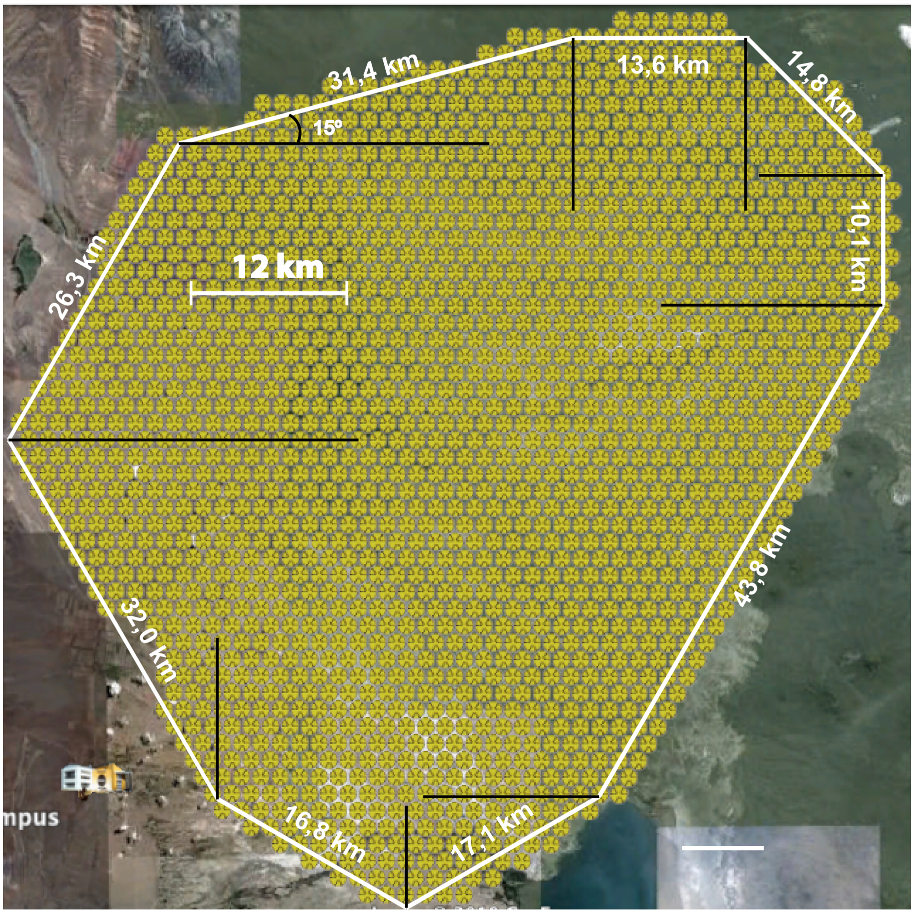


Figure 7: View of the tanks in the Pierre Auger Observatory. The length and direction of the borders is approximated for simplicity of the calculation. The size of the tanks is not to scale.

3.2) The charged particles reaching ground are detected only if they pass through the tanks. Compute the area of a tank and, using the approximate number of tanks in the Observatory given in the guide, estimate the total area covered by tanks.
3.3) In view of the very large difference between these two results, why can we still say that the effective area of the Observatory is of the order of 3000 km^2?
3.4) The number of tanks in the Observatory is a trade-off between the total cost of the

project in view of its science goals (taking into account the cost of a tank). To increase the area of the observatory without additional costs, one could think of increasing the distance between tanks. What would be the effect of doubling the distance between tanks? Which would be the new Observatory area and how many 10^{20} eV events could we expect in one year?

3.5) In this situation, what would happen at the lower energies? What aspects determine the minimal shower energy that a cosmic ray observatory is able to measure?

3.6) If the shower core is outside the Observatory area, the nearest tanks may still detect a large number of particles. However, these data are not used. Why?

4) How do we measure extreme energy cosmic rays

In this part of the work we will use the information directly measured by the Auger surface detector (tanks), which is given on the public data page individually for each shower. [After choosing a shower, move down until the end of the page containing the shower info. There you find the link “Download ASCII data for event”. These are the data you will need in this section].

The goal is to understand how, starting from this information (obtained detecting at ground level a tiny fraction of the shower particles), we can get to the characteristics of the primary particle, which interacted at the top of the atmosphere originating the shower. More specifically, we will try to discover how can we use the measurements made by each tank to estimate the energy and the direction of the primary particle.

A shower develops around an axis (corresponding to the direction of the primary particle) and the shower particles travel at speeds very close to the speed of light. The shower thus propagates in the atmosphere as a sort of disk of particles (the shower front) moving nearly at the speed of light in vacuum.

The shower core is the intersection between the shower axis and the ground: the point where the primary particle would hit the ground if there was no shower development.

The arrival direction of the showers is usually described in spherical coordinates. The relevant angles are θ, the angle of the shower axis with the vertical of the shower core, and ϕ, the angle between the horizontal projection of the shower axis and a reference direction. Schematically:

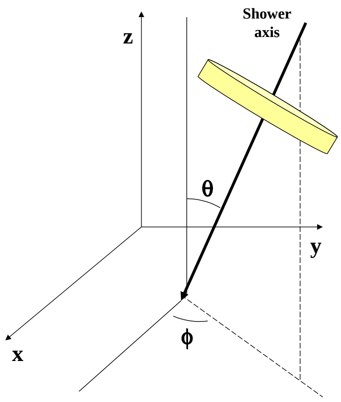


Figure 8: “Instantaneous” picture of the shower development, with the definition of the usual coordinate system.

Questions:

4.1) Which quantities are directly measured by each of the Cherenkov tanks which participate in the shower detection? Which additional information about these tanks is given?
[Note: use the ASCII files that can be obtained from the public data site in the way explained above]
4.2) Looking at the schematic representation of the arrival to the detector of showers with different directions (figure 9), which of the measured quantities contains information on the inclination of the shower? Using the shower selection tool available on the page, start by comparing rather inclined showers with nearly vertical ones (θ close to 0).

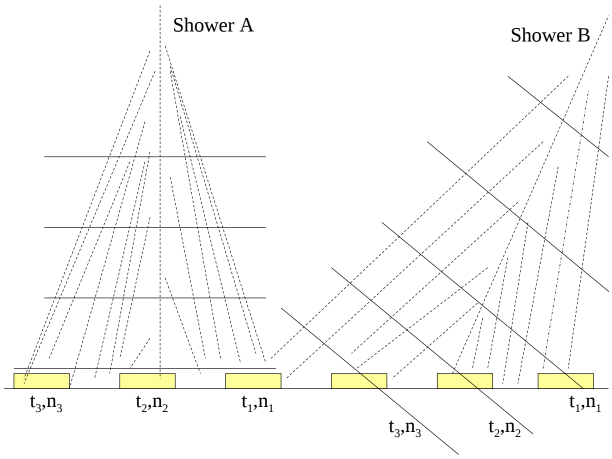


Figure 9: Schematic representation of showers with different inclinations hitting the Auger tanks.

With some trigonometry, one can obtain a simple expression for the relation between the angle θ and the quantities measured at the stations.
Perform the calculations for the following showers - showers with only 3 stations and a reasonable amount of particles detected in each of them were chosen. Use the two stations (ID1 and ID2) with the most different arrival times (the first and the last time). For each station, you will need the time and position information. Note that the shower front moves practically at the speed of light ($c = 3 \times 10^8$ m/s or $c = 0.3$ m/ns).
Fill in the table with your results. Compare them with the angle θ given in the public data page.

Event	ID1	ID2	Δt (ns)	Dist (m)	$\theta_{\text{calc}}(^\circ)$	$\theta_{\text{given}}(^\circ)$
8677500						
4796100						

4.3) For the two events used above, perform now the calculations using the combinations of stations suggested in the table below. Fill in the table.

Analysing the image of the shower configuration on ground (see public data web page) comment on the results obtained.
On which variable besides the time does the θ calculation depend?

Event	ID1	ID2	Δt (ns)	Dist (m)	$\theta_{\text{calc}}(^\circ)$	$\theta_{\text{given}}(^\circ)$
8677500	411	413				
4796100	508	543				

4.4) Using the quantities measured for each station and its position coordinates (Northing and Easting, Cartesian coordinates in meters with respect to a given reference point), build variables which give a rough estimate of the Northing and the Easting of the core (impact point) of these showers.
Compare the results obtained with those given in the public data web page. By how many meters do they differ?
4.5) Looking at the schematic representations of showers (figures 9 and 10), which are the two characteristics of the shower that affect the number of tanks in the event? Discuss how varying each of them alone changes the number of stations with signal.

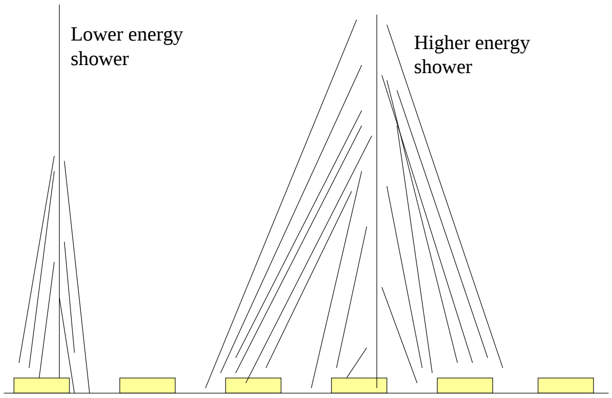


Figure 10: Schematic representation of showers with different energies and similar directions hitting the Auger tanks.

4.6) For the 3 highest energy showers available in the public data web page, let's have a look at the lateral shower profile.
For these events, which is the maximum distance to the core of a station with signal?
And what is the ratio between the signals in the most distant station and in the station with the highest signal?
Check these answers using the lateral shower distribution graphs given in the page. Looking at these graphs and at the shower configuration on ground (also shown in the page), try to estimate the detection threshold of the tanks (the minimum signal that will be registered) and the approximate size of the cascade on ground.

5) What these data tell us about extreme energy cosmic rays

In this final part of the work, you will answer some general questions about the extreme energy cosmic rays measured in Auger. You will use the ASCII file with information about all showers available from the public data web page to make the graphs you think are more adequate to be able to answer these questions.
In some cases, these graphs will be histograms or frequency graphs of a given variable: we divide the possible values of a variable in intervals; then we see how many events "fall" within each interval. In the Auger education page there are detailed instruction on how to make an energy histogram. If the concept is not familiar to you, just follow the link "How to make an energy histogram".

Questions:

5.1) Consider all showers with energies above 3 EeV. Which directions do these cosmic rays come from? Are they isotropic (do they have the same probability of coming from any direction)? Make an histogram for each of the variables θ, $\cos\theta$ and ϕ.
5.2) What are these directions in the sky? Do they come from the Sun? Which factor determines (and limits) the directions in the sky seen by the Pierre Auger Observatory in Argentina? Find the graph published by Auger and compare it with your results.
[**Hint:** To go from the local direction to galactic coordinates, we must use also the time information. Details are given in the Auger Education website.]
5.3) In order to establish the relation between the number of tanks and the shower energy, represent it graphically for all these showers. Divide the sample in two or three $\cos\theta$ regions.
5.4) What are the energies of these cosmic rays? Make an energy histogram. Compare with the energy spectrum you already know (see figure 1) and comment on what you observe.
In the intermediate energy range, are your results compatible with the conclusions of section 1? At lower energies, how do you explain the decrease of the number of events detected as the energy decreases?
Multiply the spectrum by E^3. Find the graph published by Auger and compare it with your results.
5.5) Consider the forces acting on a particle of energy E and charge Z, describing a trajectory under a magnetic field B, that projects in the plane perpendicular to the field B as a circle of radius R, and relate these quantities.
[**Hint:** $E^2 = mc^2 + p^2c$, but since $E >> mc^2$, $p \sim E/c$].
Assuming the magnetic field within a galaxy is of the order of 1 μGauss, compute the radius of curvature for a 10 PeV proton and for a 10 EeV proton. What would it be for iron nuclei of the same energies?
5.6) Taking into account the previous result and figure 11, would you say that the cosmic rays detected by the Pierre Auger Observatory are of galactic origin, extragalactic origin,

or both? For the last possibility, can you identify in the public data set an energy value where the transition may happen?

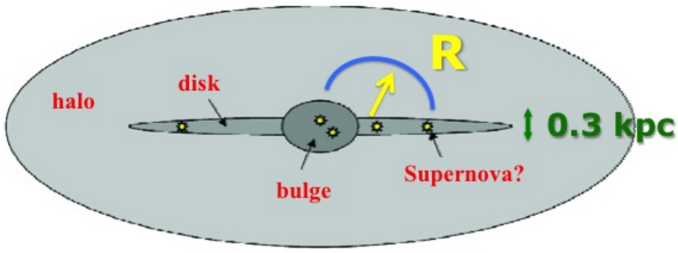


Figure 11: Schematic representation of our galaxy.

5.7) For a cosmic particle crossing the intergalactic space, we can no longer assume a constant and coherent (always in the same direction) magnetic field B. The total deviation of the direction of the particle is the result of many small "kicks" in different directions (caused by much weaker magnetic fields). Even though the Pierre Auger Observatory is able to reconstruct the primary arrival directions with an uncertainty lower than $1°$, the directions of the cosmic particles arriving from the same source would be reconstructed with differences larger than $1°$.
Assuming that 10^{20} eV protons deviate by about $3°$ when travelling from some Galaxy to Earth, what would be the deviation for iron nuclei of the same energy in the same path? And what energy would an iron nucleus need to have for its deviation a value not larger than $3°$?
Find the results published by the Pierre Auger Observatory for the arrival directions of the very highest energy particles - which cannot be found in the public data set - and observe the clustering of cosmic ray arrival directions in the vicinity of the direction of the nearest AGN (Active Galactic Nuclei, Centaurus A, about 14×10^6 light-years away).

Appendix: proposed answers

This appendix complements the exploratory guide proposed above and is organized in the same manner. It is meant as a guideline for the tutor or teacher accompanying the students exploring the guide. The answers proposed were prepared for the non-expert, although a bit of basic math is necessary to follow the arguments. Most results are approximate, and sometimes a more thorough and rigorous mathematical treatment would be required, that lies outside the scope of this guide.

1) The cosmic ray spectrum

1.1) The Avogadro constant is the number of constituent particles [1] in a sample per mol of that substance, $N_A = 6.02 \times 10^{23}$ mol^{-1}.

Making the approximations that:

i) only protons, neutrons and electrons are present in an apple;

ii) the weight of electrons is negligible (0.05% of the weight of neutrons and protons;

iii) the molar mass constant for protons and neutrons is the same and equal to $1\ u = 1$ g/mol.

we can calculate the total number of nucleons (protons+neutrons) as:

$$N_{p+n} = \frac{100[\mathrm{g}]}{1[\mathrm{g/mol}]} \times 6.02 \times 10^{23}[\mathrm{mol}^{-1}] = 6.02 \times 10^{25} \tag{1}$$

So, inside an apple, the protons have an average kinetic energy of

$$\langle E \rangle = \frac{10^{19}}{6.02 \times 10^{25}}[\mathrm{eV}] = 1.66 \times 10^{-7}\ \mathrm{eV} \tag{2}$$

which means the kinetic energies of the two protons are 26 orders of magnitude apart!

1.2) From A to B the differential flux decreases 12 orders of magnitude (from 10^{-1} to 10^{-13} $[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1}$) while the energy only increases 4.5 orders of magnitude (from 10^2 to $10^{6.5}$ GeV), giving a slope of $-12/4.5 = -2.6(6)$.

[1] particles that cannot be broken into smaller pieces at the scale of energy $k \times T$ involved in the process - mostly protons, neutrons and electrons in normal pressure and temperature conditions

From B to C the differential flux decreases 10 orders of magnitude (from 10^{-13} to 10^{-23} $[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1}$) while the energy increases 3 orders of magnitude (from $10^{15.5}$ to $10^{18.5}$ GeV), giving a slope of $-10/3 = -3.3(3)$.

So, the slope is very steep, with the flux decreasing in average three orders of magnitude per unitary increase in log 10(Energy)

Considering the flux a straight line in log-log scale, its slope is given by

$$\frac{\Delta \log_{10}(\text{Flux})}{\Delta \log_{10}(\text{Energy})} = \frac{-23-(-1)}{9.5-2} = -2.9(3) \tag{3}$$

1.3) To calculate the number of particles per unit of area and time we need the angular acceptance of the observatory (π sr), and the energy and differential flux given in figure 1. So, the general formula is

$$\text{Flux}[\mathrm{m}^2\mathrm{s}]^{-1} = \text{Differential flux}[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1} \times \pi[\mathrm{sr}] \cdot \text{Energy}[\mathrm{GeV}] \tag{4}$$

A) For Energy=10^2 GeV, Differential flux=$10^{-1}[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1}$, and so

$$\text{Flux}[\mathrm{m}^2 \cdot \mathrm{s}]^{-1} = 10^{-1} \times 10^2 \times \pi[\mathrm{m}^2 \cdot \mathrm{s}]^{-1} \approx 30[\mathrm{m}^2 \cdot \mathrm{s}]^{-1} \tag{5}$$

B) For Energy= 3×10^6 GeV, Differential flux= $10^{-13}[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1}$, and so

$$\text{Flux}[\mathrm{m}^2 \cdot \text{year}]^{-1} = 10^{-13} \times 3 \times 10^6 \times \pi \times \frac{3600 \times 24 \times 365.25\mathrm{s}}{1 \text{ year}} [\mathrm{m}^2 \cdot \text{year}]^{-1} \approx 30\ [\mathrm{m}^2 \cdot \text{year}]^{-1} \tag{6}$$

C) For Energy= 3×10^9 GeV, Differential flux= $10^{-23}[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1}$, and so

$$\text{Flux}[\mathrm{km}^2 \cdot \text{year}]^{-1} = 10^{-23} \times 3 \times 10^9 \times \pi \times \frac{3.17 \times 10^7\mathrm{s}}{1 \text{ year}} \times \frac{10^6\mathrm{m}^2}{1\ \mathrm{km}^2} [\mathrm{km}^2 \cdot \text{year}]^{-1} \approx 3[\mathrm{km}^2 \cdot \text{year}]^{-1} \tag{7}$$

1.4) At 1 TeV, the differential flux is $10^{-4}[\mathrm{m}^2 \cdot \mathrm{s} \cdot \mathrm{GeV} \cdot \mathrm{sr}]^{-1}$. With an effective acceptance of 0.5 m^2sr, the number of particles expected in a year is

$$N_p[\text{year}]^{-1} = 10^{-4} \times 10^3 \times 0.5 \times \frac{3.16 \times 10^7\mathrm{s}}{1 \text{ year}} = 1.58 \times 10^6\ [\text{year}]^{-1} \tag{8}$$

So, AMS measures approximately 1.6 million particles of energy 1 TeV per year!

For 10^{19} eV showers, the flux through a detector with π sr acceptance is

$$\text{Flux}[\text{m}^2 \cdot \text{year}]^{-1} = 10^{-24} \times 10^{10} \times \pi \times \frac{3.16 \times 10^7 \text{s}}{1 \text{ year}} = 10^{-6} \; [\text{m}^2 \cdot \text{year}]^{-1} \tag{9}$$

so, the area required to measure 100 particles per year is

$$A_{req} = \frac{100}{10^{-6}}[\text{m}^2] = 100 \text{ km}^2 \tag{10}$$

As for 10^{20} eV showers, the flux in the same detector is

$$\text{Flux}[\text{m}^2 \cdot \text{year}]^{-1} = 10^{-27} \times 10^{11} \times \pi \times \frac{3.16 \times 10^7 \text{s}}{1 \text{ year}} = 10^{-8} \; [\text{m}^2 \cdot \text{year}]^{-1} \tag{11}$$

so, the area required to measure 100 particles per year is

$$A_{req} = \frac{100}{10^{-8}}[\text{m}^2] = 10^4 \text{ km}^2 \tag{12}$$

1.5) The area of the Pierre Auger Observatory (Auger) is 3000 km^2, which is good to measure particles at 10^{19} eV, but provides very low statistics at 10^{20} eV.

Large areas can be instrumented in mainly two ways, both used in Auger:

i) Sample a very small percentage of the particles that reach the ground and use these points to find the distribution in between;

ii) Make use of the fact that excited nitrogen molecules emit light and collect these photons at ground.

1.6) From the energy A on, the flux becomes too low for satellites due to their small area, and they are not a scalable measurement tool because of their very high cost. From the energy of B on, the incident particles have enough energy to create showers that can be detected on ground at high altitudes above sea level. Between A and B, the incident particles creates showers that are too small for a considerable amount of particles to reach ground.

2) How do cosmic ray showers develop

2.1) At sea level, the pressure is 1.013×10^5 Pa, as given in figure 5. Since 1 Pa = 1 N/m^2, we can also write the pressure at sea level as

$$p_0 = 101300/9.8\ [\text{kg/m}^2] = 1.04 \times 10^4\ [\text{kg/m}^2] \tag{13}$$

So, the weight above our head at sea level is

$$W = 1.04 \times 10^4 [\text{kg/m}^2] \times 0.2 \times 0.2 [\text{m}^2] = 413.5\ \text{kg} \tag{14}$$

Repeating the same thing for the Aconcagua, at 6962 m, we have (again looking at p/p_0 in fig. 5):

$$p_{6962} = 0.48 \times 101300/9.8\ [\text{kg/m}^2] = 4.96 \times 10^3\ [\text{kg/m}^2] \tag{15}$$

So, the weight above our head at Mount Aconcagua is

$$W = 4962 \times 0.2 \times 0.2 [\text{kg}] = 198.5\ \text{kg} \tag{16}$$

In first approximation, the amount of atmosphere crossed depends only on the direction of the incident particle, in particular of its zenith angle, being the distance traversed in the atmosphere given by

$$d_{tra} = \frac{h_{atm}}{\cos(\theta)} \tag{17}$$

where h_{atm} is the height of the atmosphere.

2.2) At level n,

i) $X = n \times d$;

ii) $N = 2^n$;

iii) $E = E_0/2^n$.

2.3) At shower maximum (after a number of interaction n_{max})

i) $E = E_c$, by definition

ii) Given that the shower maximum is reached when the energy per particle becomes $E = E_c$, and assuming all particles have about the same energy, we can conclude that
$N_{max} = E_0/E_c$.

iii) In this item we shall need to estimate the number of levels n_{max} in the development of the shower to attain its maximum, which we can get from the previous information as
$n_{max} = \log_2 N_{max} = \ln(N_{max})/\ln(2)$ because $N_{max} = 2^{n_{max}}$.
Now, since $d = \lambda \times \ln(2)$

$$X_{max} = n_{max} \times d = \frac{\ln(N_{max})}{\ln(2)} \times d = \frac{\ln\left(\frac{E_0}{E_c}\right)}{\ln(2)} \times \lambda \times \ln(2) = \lambda \times \ln\left(\frac{E_0}{E_c}\right) \quad (18)$$

X_{max} increases with the logarithm of the energy of the primary particle, E_0, which is slower than the linear increase of N_{max}.

2.4) Given $E_0 = 10^{19}$ eV,

i) $N_{max} = \frac{10^{19}\text{eV}}{85\times10^{6}\text{eV}} = 1.18 \times 10^{11}$ particles

ii) $X_{max} = 36.7 \times \ln(1.18 \times 10^{11}) = 935.6\ g/cm^2$

The Pierre Auger Observatory is at a height of approximately 1400 m above sea level. Looking at figure 5 we can see that corresponds to a pressure of about $0.86 \times p_0$, and so the observatory value of X is

$$X[\text{g/cm}^2] = 0.86 \times \frac{101300\ \text{N/m}^2}{9.8\ \text{m/s}^2} = 8889.6\ \text{kg/m}^2 = 889.0\ \text{g/cm}^2. \quad (19)$$

We can thus see that in Auger many 10^{19} eV showers reach the ground before reaching their maximum number of particles.

2.5) Examples of the assumptions/approximations that are made in the Heitler model and may not hold are:

- d is not a number but a distribution. Also, this distribution is not the same for photon emission and pair creation (and even in each of this processes depends on many other factors)
- electrons interact in more ways than just by emission of photons

- in the case that they emit a photon, the energy is in general not equally distributed between the electron and the photon
- the primary cosmic ray is in general not a photon, so the shower doesn't have only an electromagnetic component

2.6) Firstly, we fill the table, noting that in level n,

i) $N_{tot} = 30^n$;

ii) $N_{ch} = 20^n$;

iii) $E = \frac{E_0}{30^n}$;

iii) $E_{ch} = \left(\frac{2}{3}\right)^n \times E_0$.

The number of interactions after which charged pions decay is given, as a function of E_{dec} and E_0, by

$$E_{dec} = \frac{E_0}{30^{n_{dec}}} \Leftrightarrow n_{dec} = \frac{\ln\left(\frac{E_0}{E_{dec}}\right)}{\ln(30)} \tag{20}$$

The number of muons is then given by

$$N_\mu = 20^{n_{dec}} = 20^{\frac{\ln\left(\frac{E_0}{E_{dec}}\right)}{\ln(30)}} = \left(20^{\log_{20}\left(\frac{E_0}{E_{dec}}\right)}\right)^{\frac{\ln(20)}{\ln(30)}} = \left(\frac{E_0}{E_{dec}}\right)^{\frac{\ln(20)}{\ln(30)}} \tag{21}$$

2.7) The energy transferred to the electromagnetic shower is equal to the total energy minus the energy in charged pions in level n, so when the energy per particle reaches E_{dec}, the fraction of energy in the electromagnetic shower is

$$\frac{E_{el}}{E_0} = 1 - \left(\frac{2}{3}\right)^{n_{dec}} = 1 - \left(\left(\frac{2}{3}\right)^{\log_{2/3}\left(\frac{E_0}{E_{dec}}\right)}\right)^{\frac{\ln(2/3)}{\ln(30)}} = \left(\frac{E_0}{E_{dec}}\right)^{\frac{\ln(2/3)}{\ln(30)}} \approx \left(\frac{E_0}{E_{dec}}\right)^{-1.192} \tag{22}$$

For 10^{19} eV showers,

$$\frac{E_{el}}{E_0} = \left(\frac{E_0}{E_{dec}}\right)^{-1.192} = \left(\frac{10^{19}\text{ eV}}{20 \times 10^6}\right)^{-1.192} = 1 - 0.041 \approx 96\% \tag{23}$$

2.8) For a proton shower

i)

$$X_{max} = d \times n_{dec} = d \times \frac{\ln\left(\frac{E_0}{E_{dec}}\right)}{\ln(30)} \tag{24}$$

ii)

$$N_{max} = 30^{n_{dec}} = \frac{E_0}{E_{dec}} \tag{25}$$

iii)

$$N_\mu = \left(\frac{E_0}{E_{dec}}\right)^{\frac{\ln(20)}{\ln(30)}} \tag{26}$$

For iron showers, considering the superposition model, of 56 protons each with an energy $E_0/56$

i)

$$X_{max} = d \times n_{dec} = d \times \frac{\ln\left(\frac{E_0/56}{E_{dec}}\right)}{\ln(30)} = d \times \frac{\ln\left(\frac{E_0}{E_{dec}}\right)}{\ln(30)} - d \times \frac{\ln(56)}{\ln(30)} \tag{27}$$

ii)

$$N_{max} = 56 \times 30^{n_{dec}} = 56 \times 30^{\frac{\ln\left(\frac{E_0/56}{E_{dec}}\right)}{\ln(30)}} = 56 \times \frac{E_0/56}{E_{dec}} = \frac{E_0}{E_{dec}} \tag{28}$$

iii)

$$N_\mu = 56 \times 20^{n_{dec}} = 56 \times \left(\frac{E_0/56}{E_{dec}}\right)^{\frac{\ln(20)}{\ln(30)}} = 56^{1-\frac{\ln(20)}{\ln(30)}} \times \left(\frac{E_0}{E_{dec}}\right)^{\frac{\ln(20)}{\ln(30)}} \tag{29}$$

So, the difference between X_{max} for proton and iron showers is

$$X_{max}(p) - X_{max}(Fe) = d \times \frac{\ln(56)}{\ln(30)} \approx 1.18 \times d, \tag{30}$$

the maximum number of particles is the same in both cases.
The ratio between the number of muons for both particle showers is

$$\frac{N_\mu(Fe)}{N_\mu(p)} = 56^{1-\frac{\ln(20)}{\ln(30)}} \approx 1.616 \tag{31}$$

So, we can conclude that since irons have more particles interacting at the beginning, they develop faster into a shower, having less levels (n) and therefore creating more muons.

2.9) From figure 4, the radius of the shower is around 2500 m, so the area it covers on ground is approximately $\pi \times (2.5\ \mathrm{km})^2 \approx 20\ \mathrm{km}^2$

A muon produced at an height of 10 km and that reaches the ground at 2.5 km from the core has an angle with respect to the shower axis of

$$\theta = \arctan\left(\frac{2.5}{10}\right) \approx 14^o \tag{32}$$

2.10) Since, as we have already seen, $\lambda = 36.7\ \mathrm{g/cm^2}$ and $E_c = 85$ MeV,

$$r_M\ [\mathrm{g/cm^2}] = \frac{21\mathrm{MeV}}{85\mathrm{MeV}} \times 36.7\ \mathrm{g/cm^2} = 90.7\ \mathrm{kg/m^2} \tag{33}$$

Since the air density at 1400 m is $0.86 \times \rho_0$

$$\rho_{1400} = 0.86 \times 1.2\ \mathrm{kg/m^3} = 1.056\ \mathrm{kg/m^3} \tag{34}$$

and

$$r_M\ [\mathrm{m}] = \frac{r_M\ [\mathrm{kg/m^2}]}{\rho_{1400}[\mathrm{kg/m^3}]} = \frac{90.7}{1.056}\ \mathrm{m} \approx 86\ \mathrm{m} \tag{35}$$

So, although the total shower radius is very large, the vast majority of its energy is concentrated very close to the core.

3) How do we detect cosmic ray showers

3.1) The area covered by the Observatory can be computed as the sum of areas of triangular and rectangular regions.These regions are defined from the map by prolonging the horizontal and vertical lines. Note that the area of a triangle with a $90°$ angle can be computed by $A = \frac{hyp^2}{4} \times \sin(2\alpha)$, with hyp being the size of the hypotenuse and α being one of the smaller angles, which can be always $30°$ except in the top left triangle that it can be $15°$.
With this in mind we got the approximate value of $A_{tot} = 2970$ km^2.

3.2) The diameter of each tank is 3.6 m, so its area is $\pi \times 1.8^2 = 10.17\text{m}^2$. Since the number of tanks is approximately 1600, the total area occupied by tanks is

$$A_{\text{tanks}} = 10.17 \times 1600 \text{ m}^2 = 16272 \text{ m}^2 = 0.0163 \text{ km}^2 \quad (36)$$

So the tanks only cover about $0.0163/3000 = 5.4 \times 10^{-6} = 0.00054\%$ of the area of the Observatory.

3.3) As we have seen, high energy showers have radius of the order of 2.5 km, so we only need to have tanks with small enough spacing so that the shower, when it reaches the ground, hits a sufficient number of tanks to allow the reconstruction of the properties of the shower.
In the Pierre Auger Observatory, the spacing of 1.5 km was chosen so that most high energy showers are seen by at least 5 tanks (in a simplistic view, in a circle of diameter 5 km one can put 3 tanks in each axis).

3.4) If we double the distance between tanks, we would get a factor of 2 in both the width and the length of the Observatory, and thus a factor of 4 in the total area. The new area of the Auger would be 12000 km^2.
The flux at 10^{20} eV has been calculated in 1.4) to be

$$10^{-8}[\text{m}^2 \cdot \text{year}]^{-1} = 10^{-8} \times 10^6[\text{km}^2 \text{ year}]^{-1} = 0.01[\text{km}^2 \text{ year}]^{-1} \quad (37)$$

So the expected number of particles in a year would be

$$0.01[\text{km}^2 \cdot \text{year}]^{-1} \times 12000 \text{ km}^2 = 120 \text{ particles per year} \quad (38)$$

3.5) To reconstruct showers we need to have a minimum of three tanks hit, with the reconstruction improving substantially when 5 or more tanks are used. Since the radius of the shower scales with energy, when we increase the distance between tanks we are decreasing the number of tanks that are hit by particles in a given event, and below a given energy we will stop having enough tanks to be able to

reconstruct events.
Therefore, the effect of increasing the spacing between tanks is, on the positive side, the increase of the area and thus of statistics at high energies, but on the flip side the increase of the energy threshold, meaning that showers with lower energies cannot be measured. Together with the distance between tanks, also the altitude of the observatory is relevant. Higher altitude observatories are better for lower energies, and lower altitude for higher energies, so that the shower maximum is close to ground.

3.6) If the core is not contained within the array, we might see signal in a few stations but the problem will be poorly constrained. For example, a nearby low energy shower and a distant higher energy one could lead to similar signals. How to tell one from the other? Once we see only one side of the shower and don't know the location of the core, it is hard to reconstruct properly the shower.

4) How do we measure extreme energy cosmic rays

4.1) The quantities directly measured by the Cherenkov tanks are the times of arrival of the particles and the total signal (in VEM [1]). The other quantities, namely easting, northing and altitude, don't depend on the detection and are known for every tank, independent of the signal.

4.2) Looking at figure 9, we can see that it is possible to calculate the angle θ with the vertical axis based on the timing information of the tanks. Being l the distance traversed by the shower front between hitting each tank, the angle is given by

$$\sin(\theta) = \frac{l}{d_{\text{tanks}}} \Leftrightarrow \theta = sin^{-1}\left(\frac{0.3[\text{m/ns}] \times \Delta t[\text{ns}]}{1500[\text{m}]}\right) \tag{39}$$

where Δt [ns] is the difference in time between stations. Note that you also don't have to take the distance as 1500 m, you can calculate it with the differences in Easting and Northing provided in the file. Doing this, the table can be filled

Event	ID1	ID2	Δt [ns]	Dist [m]	$\theta_{calc}[^o]$	$\theta_{given}[^o]$	$\Delta\theta[^o]$
8677500	418	413	3585	1498.96	45.85	46.7 ± 0.7	0.85
4796100	576	508	1517	1497.95	17.69	18.0 ± 0.4	0.31

4.3) We now fill the 2^{nd} table using the same formula as above

Event	ID1	ID2	Δt [ns]	Dist [m]	$\theta_{calc}[^o]$	$\theta_{given}[^o]$	$\Delta\theta[^o]$
8677500	411	413	1967	1502.26	23.13	46.7 ± 0.7	23.57
4796100	508	543	377	1504.23	4.31	18.0 ± 0.4	13.69

Clearly, we can see that the variables calculated by our formula are not compatible with the ones given in the auger website...
When trying to understand the difference between the previous situation, in which the θ calculation yielded the expected results, and the present situation, in which it does not seem to work, we can notice the following: before, we were choosing a pair of stations along a direction approximately aligned with the projection of the shower axis on ground. Here, this is no longer true. In fact, for the same zenith

[1] 1 VEM is the signal given by a high energy muon traversing a tank vertically

angle, there are many possible directions, which correspond to different projections on ground. This is described by the azimuthal angle ϕ. The direction of the cosmic ray is given by (θ, ϕ) and they have to be computed together as they depend on each other.
Just for curious people, we leave here an approximate solution, based on the assumptions that the speed of particles is equal to c (speed of light in vacuum), and that the shower front is a very thin planar disc:

$$\begin{aligned} \tan\phi &= -\frac{d_{12}\Delta t_{23}\cos\phi_1 - d_{13}\Delta t_{12}\cos\phi_2}{d_{12}\Delta t_{23}\sin\phi_1 - d_{13}\Delta t_{12}\sin\phi_2} \\ \sin\theta &= \frac{c\Delta t_1}{d_{12}\cos(\phi-\phi_1)} = \frac{c\Delta t_2}{d_{23}\cos(\phi-\phi_2)} \end{aligned}$$

4.4) The core is in first approximation the barycentre of the tanks in the event, so it can be easily calculated by averaging the Easting and Northing coordinates of the tanks in each event, weighted by their signal in VEM.
So, for example in Excel, the Northing coordinate of the core is given by

$$Nor = SUM(Signal[1,T] \times North[1,T])/(SUM(Signal[1,T])) \tag{40}$$

where T is the total number of tanks in the event.
Look for example at the event number 4128900 (most seen event in the page as of October 2012). For this event we get

	$Core_{given}$ [m]	$Core_{calc}$ [m]	$\Delta(\lvert calc - given\rvert)$ [m]	Total difference [m]
Easting	479789	480055.6	266.65	311.68
Northing	6073404	6073565	161.38	

So we have a non negligible difference between the position of the core calculated by our method and given in the page. To understand this problem, we can look at the image of the shower on the page and see the tanks inside the ellipsis that did not have signal, and we can even estimate the relative displacement expected in the core calculation due to having less tanks above and below each axis. For showers in which all the tanks inside the ellipsis recorded a signal, repeating the same process gives very precise results.

4.5) Looking at the figures showing schematic representations of showers with different zenith angles and with different energies, we can say that both these shower characteristics change the number of tanks “hit” by the shower. If we are at fixed energies, the number of stations will increase with the inclination of the shower. Conversely, for showers with similar zenith angles (which we can select after reconstructing the direction) the number of trigered stations (or in other words the shower size on ground) gives information on the shower energy: the larger the “footprint”, the higher the energy of the shower.

4.6) As of October 2012, the 3 highest energy showers were:

- 15457900: 49.30 EeV, 19 stations, 59.4 deg, Jul 02 2012 08:09
- 10485600: 49.93 EeV, 13 stations, 40.2 deg, Oct 26 2010 17:39
- 4128900: 41.07 EeV, 18 stations, 54.6 deg, Oct 30 2007 11:14

and the signal at the closest and farthest station is respectively:

- 15457900:

$$S(400) = 1000\ [VEM]\ ; \qquad S(3800) \approx 7\ [VEM]\ ; \qquad \frac{S(3800)}{S(400)} = 7 \times 10^{-3} \tag{41}$$

- 10485600:

$$S(400) = 5000\ [VEM]\ ; \qquad S(3200) \approx 4\ [VEM]\ ; \qquad \frac{S(3800)}{S(400)} = 8 \times 10^{-4} \tag{42}$$

- 4128900:

$$S(500) = 1000\ [VEM]\ ; \qquad S(2800) \approx 3\ [VEM]\ ; \qquad \frac{S(3800)}{S(400)} = 3 \times 10^{-3} \tag{43}$$

Looking at the tanks with the lowest signal, we estimate the threshold to be around 3 VEM, and the size of the largest available shower on the ground to be around $\pi \times 4^2$ km^2 = 50.24 km^2.

5) What the public data tell us about extreme energy cosmic rays

5.1) Download the file auger_public_****_**_**.txt from the Event Display section of the Auger official website, and import it to Excel as a space separated ASCII file. Then, define in another column the binning for the histograms you need (ϕ, θ and $\cos(\theta)$ in this case).

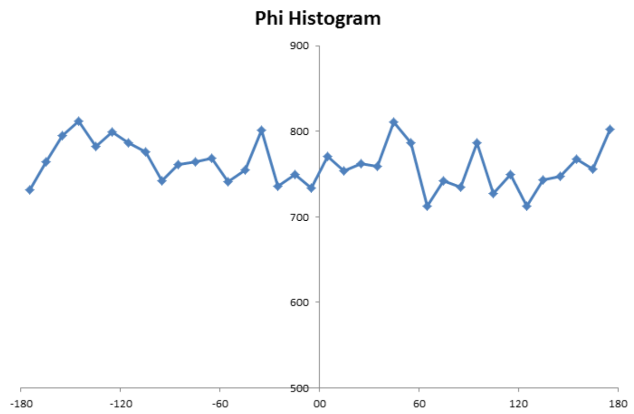


Figure 12: Histogram of ϕ for all public events.

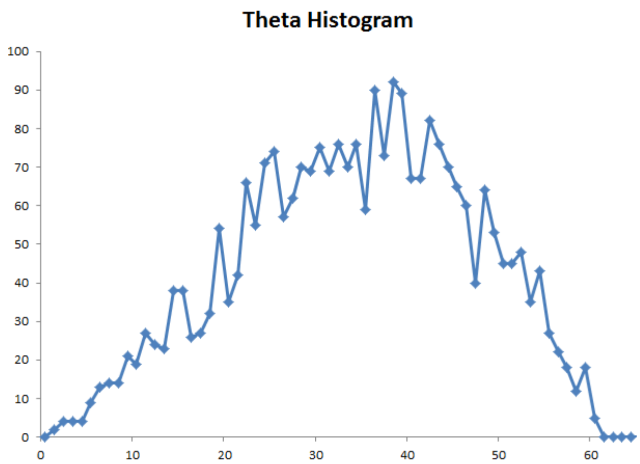


Figure 13: Histogram of θ for all public events.

Then, we can make histograms by using the Data→Data Analysis→Histogram tool in Excel, with results that should look similar to figures 12 and 13.

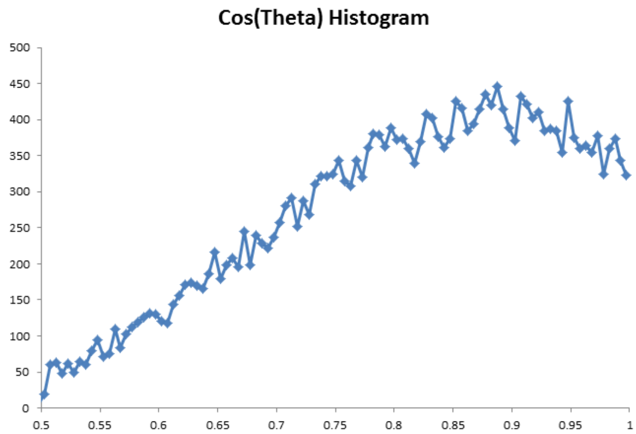


Figure 14: Histogram of $\cos(\theta)$ for all public events.

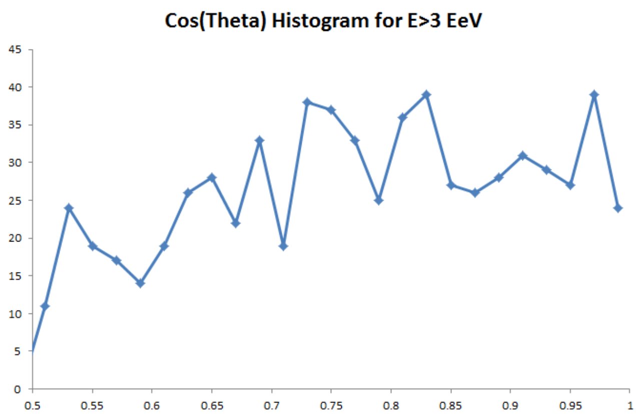


Figure 15: Histogram of $\cos(\theta)$ for events with $E > 3$ EeV.

Note that while the flux is constant in ϕ, it is not in θ. If we look at the definition of solid angle, we can see that it depends linearly on ϕ and on $\cos(\theta)$ - in fact, it is for equal ϕ and $\cos(\theta)$ (not θ) intervals that we are looking at regions of equal size in the sky.
However, making an histogram for $\cos(\theta)$ (figure 14) we see that the flux is still not constant... Why? We should note that for energies below 3 EeV the detector is not 100% efficient (it might "see" or not "see" depending for example on how far the core "falls" from the stations) and the efficiency will depend on the inclination of the shower: inclined showers hit a larger area than vertical showers and will thus have a larger probability of being detected. More inclined showers will be measured than vertical ones. Above 3 EeV the detector is fully efficient even for vertical showers.
Repeating the $\cos(\theta)$ histogram only for showers with energy above 3 EeV (figure 15) we see that, despite the low statistics, it looks approximately constant.

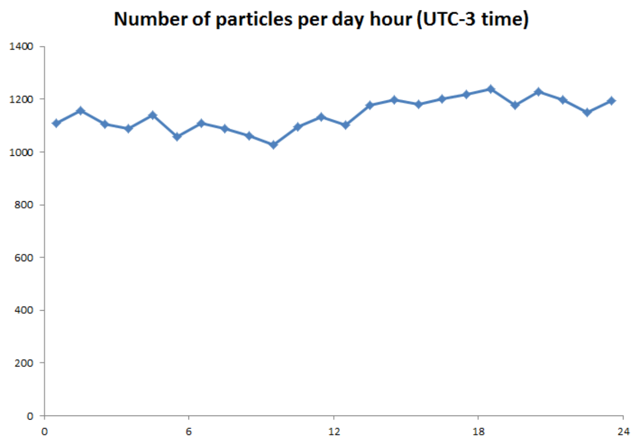


Figure 16: Histogram of the number of events per day hour in UTC-3 (Argentinean time).

5.2) To check whether cosmic rays come from the Sun, we use the Unix time given in seconds in the ASCII file, and divide it by 3600 to get the number of hours elapsed since midnight UTC January 1^{st} 1970.
Then, noting that Argentina is in UTC-3, we subtract 3 to the number of hours and make mod_{24} of the obtained value. So, in Excel, if the UNIX time is in column F, the day hour in Argentina is given by

$$h_{Arg} = MOD(F2/(3600) - 3; 24) \tag{44}$$

The histogram of the number of events as a function of the hour of the day is shown in figure 16, where it can be seen that it is approximately constant with time, invalidating the hypothesis that high energy cosmic rays come from the Sun.

The incoming directions in the sky of the cosmic rays observed by the Pierre Auger are given by the latitude and longitude coordinates (latitude is in the galactic plane, with 0^o being the galactic center, and longitude is normal to that plane with $+90^o$ being the North Pole). The scatter plot of these variables is given in figure 17, as well as the official Auger plot (figure 18).

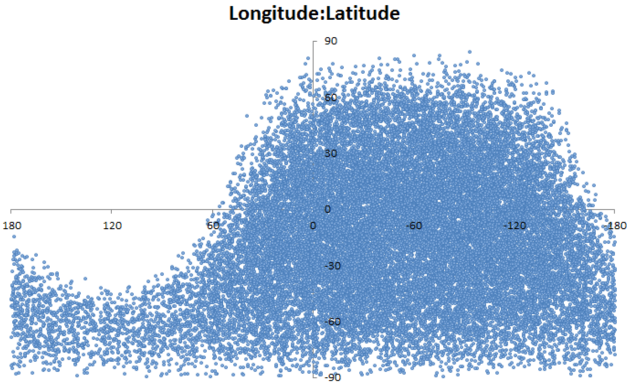


Figure 17: Latitude vs Longitude scatter plot for all Auger public events.

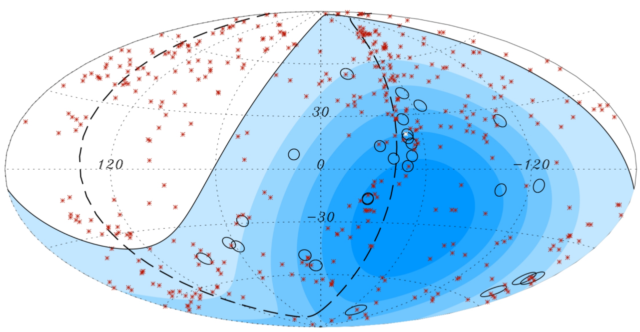


Figure 18: Official Auger visible sky plot showing also the arrival directions of the very highest energy cosmic rays (circles).

The transformation to galactic coordinates are explained in detail in the Pierre Auger Observatory Education web pages. Simply put, one needs to know two things: the time and the position on the Earth, both of which are given for the tanks. Then, we have to find the tilt of the Earth axis with respect to the galactic center (only dependent on time), as well as the rotation around that axis relative to the galactic center (different for each point on Earth and so dependent on both

the time and the position).
Finally, we define a coordinate system with the galactic plane and its normal axis, with ϕ being a rotation in the plane, with 0^o point to the galactic center, and θ the angle with respect to that plane, measured relative to the normal axis.

5.3) Plotting the logarithm of the energy as a function of the number of events we obtain figure 19. In average, the $\log 10$(Energy) increases linearly with the number of tanks with signal. However, the same number of tanks can correspond to a large range of $\log 10$(Energy) depending, for example, on the angle - as you can test. While the parameters of the fitted line have no obvious physical meaning (there is at least an important dependence on zenith angle you can test), it shows a clear positive correlation between the number of tanks and the logarithm of the energy of the cosmic ray, and that their dependence is at first approximation linear.

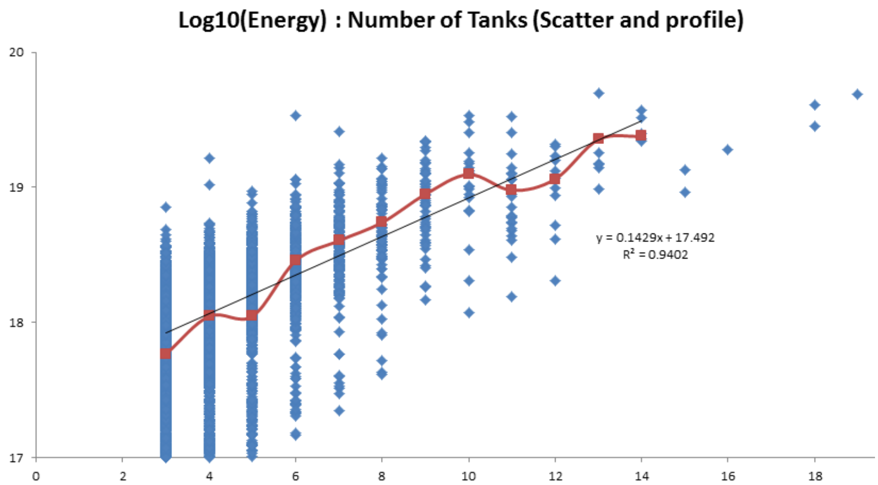


Figure 19: All values (stars) and average values (squares) of $\log 10$(Energy) vs. number of tanks. Note that the last square is the average for all events with 14 or more tanks, as only 6 event with more than 14 tanks were available.

5.4) Before doing the histograms, it should be noted that in this case variable sized bins (in EeV) may be more practical. In particular, constant bins in log10(Energy) are normally used. In this case, one should define the binning in Excel and proceed to get the histogram table as previously explained. Then, the frequency per bin has to be divided by the size of the bin (in EeV). The flux per energy can be seen in figure 20. The decrease in the lower energies is explained by the decreasing acceptance of the Observatory for energies below 10^{18} eV. If one only looks at energies above this, the fit to the flux yields the same value calculated in section

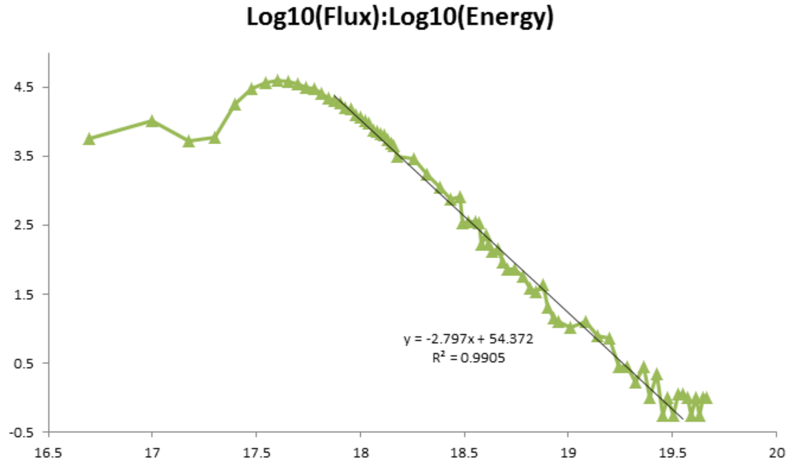


Figure 20: Log10(Flux) vs Log10(Energy).

1: a slop of approximately -2.8. We can accentuate this fact by multiplying the flux by a factor of E^3, showing an almost horizontal line for energies above 10^{18} eV. This is shown in figures 21 (this result) and 22 (official Auger result).

5.5) A charged particle moving in a magnetic field will experience a force

$$\vec{F} = q \times \left(\vec{v} \times \vec{B}\right) \tag{45}$$

Since the direction of the force is a cross-product, if the field is not collinear with the velocity, the force will always be perpendicular to both $\vec{B}$ and to the velocity, and so the projection of the particle's trajectory in the plane perpendicular to $\vec{B}$ will be circular (in space it will be an helix). The radius of the circle is determined using the centripetal force

$$\frac{m \times v_\perp^2}{R} = q \times v_\perp \times B \Leftrightarrow R = \frac{m \times v_\perp}{q \times B} \tag{46}$$

Notice that in the relativistic case we have to replace $m \times v$ with p, which is given by

$$p \times c = \sqrt{E^2 - m^2 c^4} = \sqrt{10^{38}\mathrm{eV}^2 - 10^{18}\mathrm{eV}^2} = 10^{19}\ \mathrm{eV} \approx E \tag{47}$$

assuming proton and using natural units (c=1). Considering iron nuclei would not make a difference, as one can see replacing m^2 with 56×10^9 eV instead of 10^9 eV.

For the values given, noting that 1 $\mu G = 10^{-6}\ G = 10^{-10}\ T$ and that 1 pc= 3.26 light-year = 3.10^{16} m, and that for iron $Z = 26$ (although $A = 56$) , we have

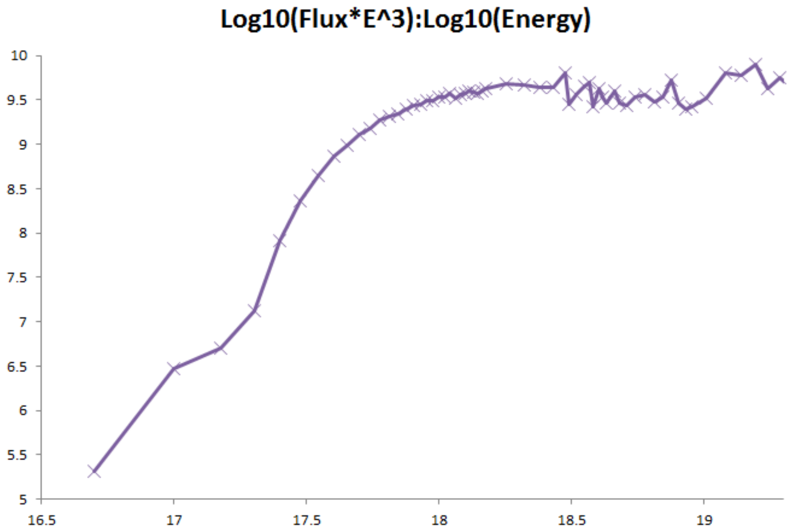


Figure 21: Log10(Flux$\times E^3$) vs Log10(Energy).

the following radius

i) 10 PeV (10×10^{15} eV) proton

$$R = \frac{10^{16}\ [\mathrm{eV/c}]}{1\ [\mathrm{e}] \times 10^{-10}\ T} = \frac{10^{26}}{3 \times 10^{8}}\ [\mathrm{m}] = 3 \times 10^{17}\ \mathrm{m} \approx \frac{3 \times 10^{17} m}{3 \times 10^{16} [\mathrm{m/pc}]}\ \mathrm{pc} = 0.01\ \mathrm{kpc} \tag{48}$$

ii) 10 EeV (10×10^{18} eV) proton

$$R = \frac{10^{19}\ [\mathrm{eV/c}]}{1\ [\mathrm{e}] \times 10^{-10}\ T} \approx \frac{3 \times 10^{20}\ \mathrm{m}}{3 \times 10^{16}\ [\mathrm{m/pc}]}\ \mathrm{pc} = 10\ \mathrm{kpc} \tag{49}$$

iii) 10 PeV iron nucleus

$$R = \frac{10^{16}\ [\mathrm{eV/c}]}{26\ [\mathrm{e}] \times 10^{-10}\ T} = \frac{10^{26}}{26 \times 3 \times 10^{8}}\ [\mathrm{m}] = 1.1 \times 10^{16}\ \mathrm{m} \approx 0.00039\ \mathrm{kpc} \tag{50}$$

iv) 10 EeV iron nucleus

$$R = \frac{10^{19}\ [\mathrm{eV/c}]}{26\ [\mathrm{e}] \times 10^{-10}\ T} \approx \frac{1.1 \times 10^{19}\ \mathrm{m}}{3 \times 10^{16}\ [\mathrm{m/pc}]}\ \mathrm{pc} = 0.39\ \mathrm{kpc} \tag{51}$$

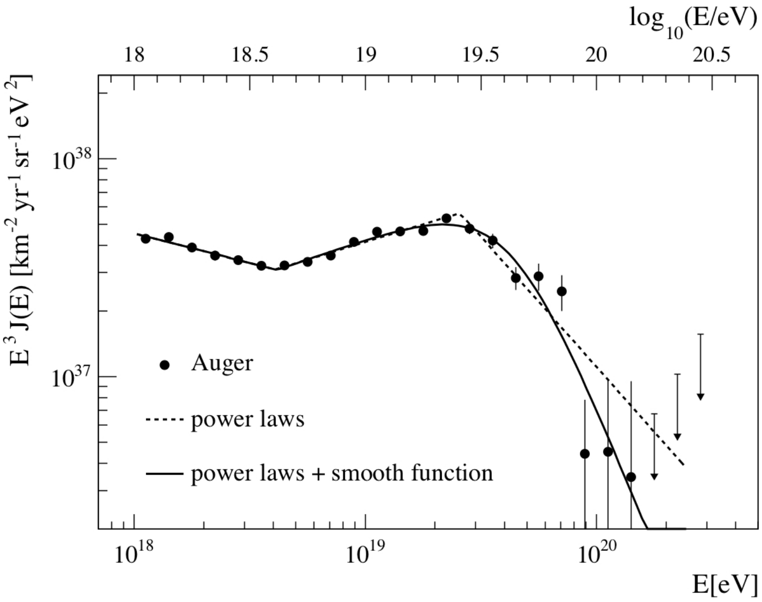


Figure 22: Official Auger energy spectrum plot.

5.6) Considering a galaxy disc with a thickness of 0.3 kpc, we see that 10 EeV iron nuclei can have a larger radius of curvature. Protons of the same energy, with a 10 kpc radius, certainly arrive to us from other galaxies. At 10 PeV, both particles are contained within the galactic disc and so the cosmic rays that reach us can be galactic.

5.7) The deviation of cosmic rays from their original trajectory comes from a series of small deviations suffered while crossing the numerous different magnetic fields (both in magnitude and in direction) in their path. At each moment in time, the magnitude of the deflection is proportional to Z. For iron nuclei, we thus have a radius of about $26 \times 3° = 78°$.